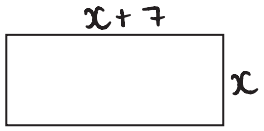
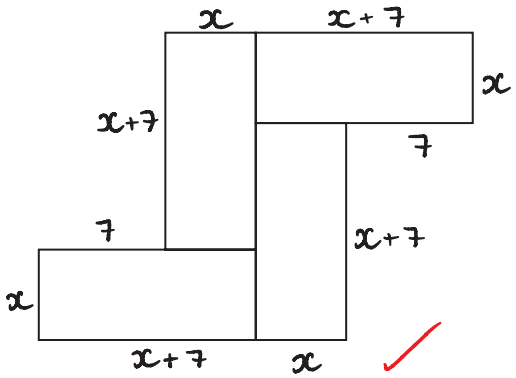
1. Here is a rectangle.



The length of the rectangle is 7 cm longer than the width of the rectangle.

4 of these rectangles are used to make this 8-sided shape.



x + 7 − x = 7

The perimeter of the 8-sided shape is 70 cm.

Work out the area of the 8-sided shape.

Let x be width of the rectangle

$70 = x+7+x+7+x+7+x+x+7+x+7+x+7+x$

$70 = 8x + 42$

$(-42) \quad (-42)$

$28 = 8x$

$(\div 8) \quad (\div 8)$

$3.5 = x$

Area of rectangle = width × length

$= x(x+7)$

$= 3.5 \times (3.5 + 7)$

$= 3.5 \times 10.5$

$= 36.25$

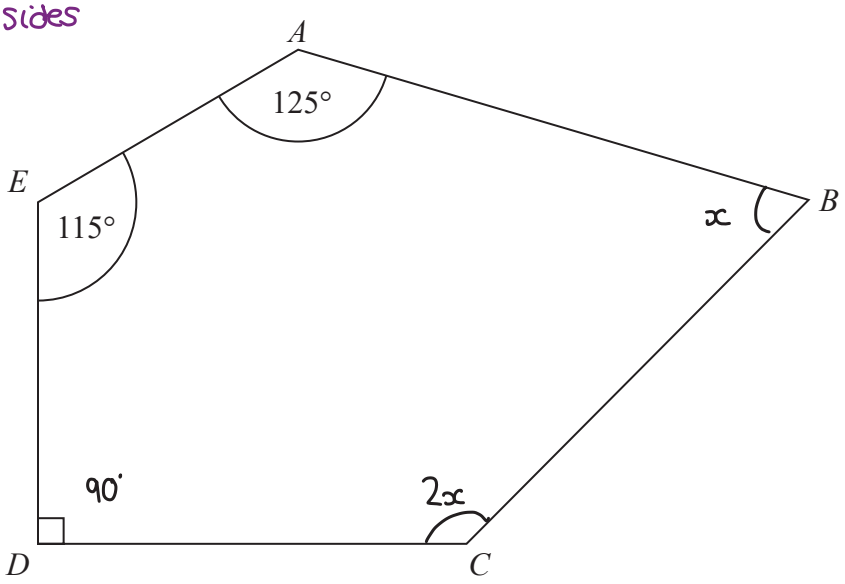
$36.25 \times 4 = 147 \text{ cm}^2$

147 cm²

(Total for Question is 5 marks)

2. *ABCDE* is a pentagon.

5 sides



Angle *BCD* = 2 × angle *ABC*

Work out the size of angle *BCD*.
You must show all your working.

Let $\angle ABC = x$ $\therefore$ $\angle BCD = 2x$

Sum of interior angles of a pentagon:

number of sides

$(n-2)\times 180 = (5-2)\times 180$ (1)

$= 180\times 3$

$= 540°$ (1)

Setting up an equation in x:

$x + 2x + 90 + 115 + 125 = 540$ (1)

$3x = 210$ (1)

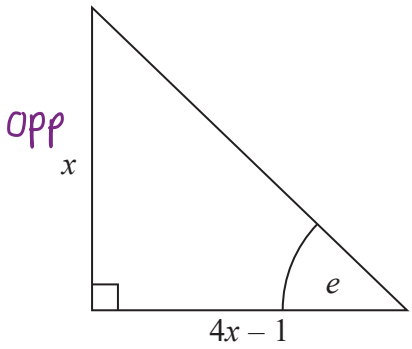
$x = 70°$

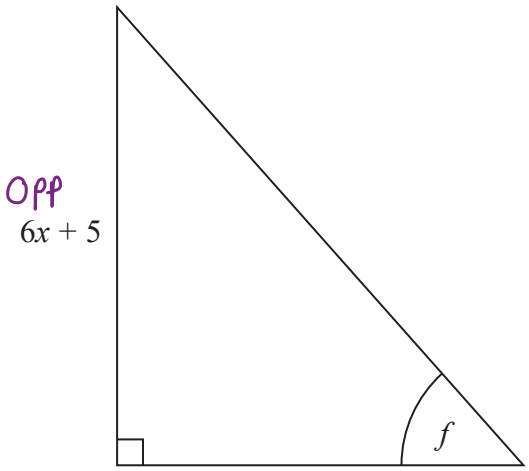
$\angle BCD = 2x = 2\times 70 = 140°$

140 (1) °

(Total for Question is 5 marks)

3. Here are two right-angled triangles.





Given that

$\tan e = \tan f$

SOH CAH TOA

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

find the value of x.

You must show all your working.

$\tan e = \frac{x}{4x-1}$ $\quad \tan f = \frac{6x+5}{12x+31}$

Cross multiply

$\frac{x}{4x-1} = \frac{6x+5}{12x+31}$ (1)

$x(12x+31) = (6x+5)(4x-1)$ Multiply out (1)

$12x^2 + 31x = 24x^2 - 6x + 20x - 5$

$12x^2 + 31x = 24x^2 + 14x - 5$

$0 = (24x^2 - 12x^2) + (14x - 31x) - 5$ $\quad -(12x^2 + 31x)$

$0 = 12x^2 - 17x - 5$ (1)

Solving for x :

(By factorisation or using the quadratic formula)

$(4x+1)(3x-5) = 0$ (1)

$\therefore$ either $4x+1=0$ or $3x-5=0$

$4x = -1$ $\quad 3x = 5$

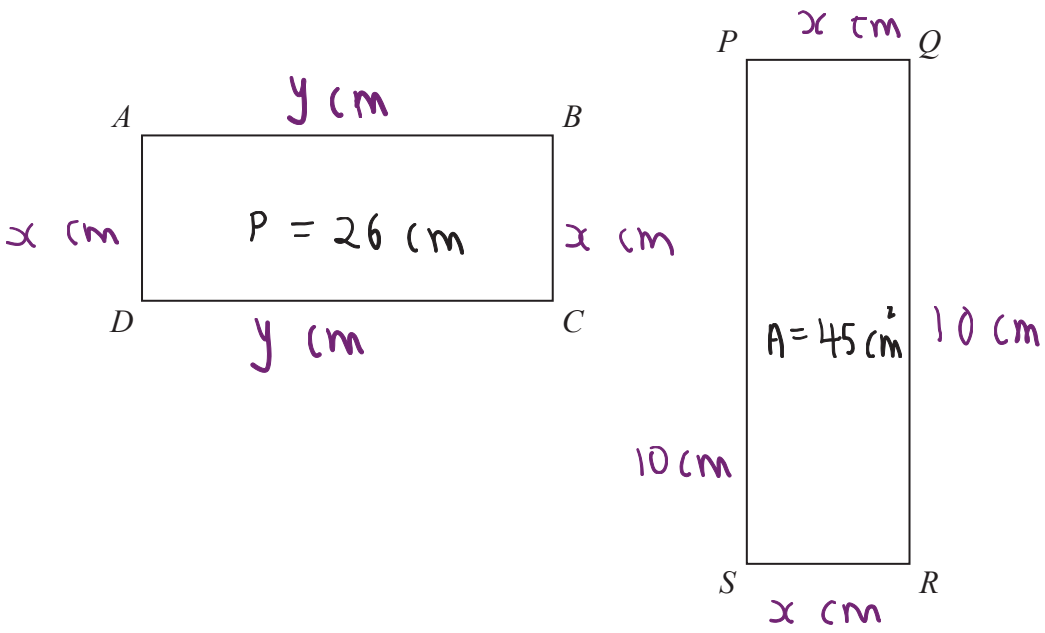
$x = -\frac{1}{4}$ $\quad x = \frac{5}{3}$

$x > 0$ as it is a length so this solution is not valid

$\frac{5}{3}$ (1)

(Total for Question is 5 marks)

4. Here are two rectangles.



$QR = 10$ cm
$BC = PQ$

The perimeter of $ABCD$ is 26 cm
The area of $PQRS$ is 45 cm²

Find the length of AB.

$ABCD: \quad 2x + 2y = 26$

$PQRS: \quad x(10) = 45$ (1)

$10x = 45$

$\div 10$ $\quad x = 4.5$ $\quad \div 10$

(1)

$2(4.5) + 2y = 26$

$9 + 2y = 26$. (1)

$2y = 17$.

$\div 2$ $\quad y = 8.5$ $\quad \div 2$

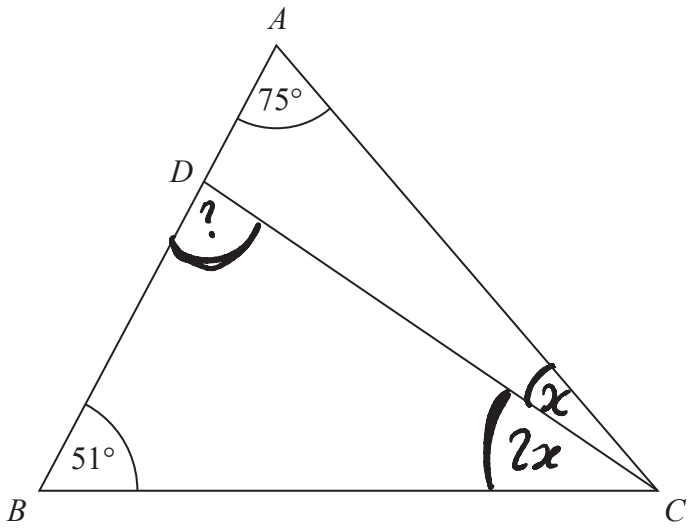
y = length AB.

Length AB = 8.5 cm. (1)

8.5 cm

(Total for Question is 4 marks)

5. The diagram shows triangle *ABC*.

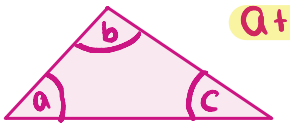


ADB is a straight line.

the size of angle *DCB* : the size of angle *ACD* = 2 : 1 ✓

Work out the size of angle *BDC*.

All interior angles of a triangle add to 180°



a+b+c = 180

For Triangle ABC

$75+51+2x+x=180$

$3x=180-75-51$

$3x=54$ (1)

$x=\frac{54}{3}$

$x=18$ (1)

For Triangle BCD

$51+2x+?=180$

Since $x=18$ (1)

$51+2(18)+?=180$

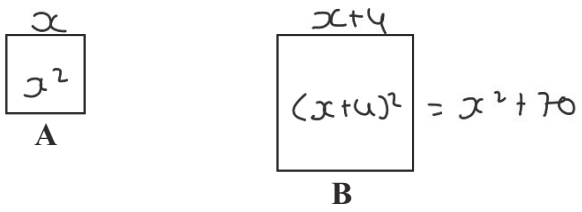
$?=180-51-2(18)$

$=180-51-36$

$=93$

(1) 93 °

6. Here are two squares, **A** and **B**.



The length of each side of square **B** is 4 cm greater than the length of each side of square **A**.
The area of square **B** is 70 cm² greater than the area of square **A**.

Find the area of square **B**.
Give your answer correct to 3 significant figures.
You must show all your working.

Area of B = $(x+4)^2$

$= \left(\frac{54}{8}+4\right)^2$

$= 116$

$(x+4)^2 = x^2 + 70$

$x^2 + 8x + 16 = x^2 + 70$

$8x = 70 - 16$

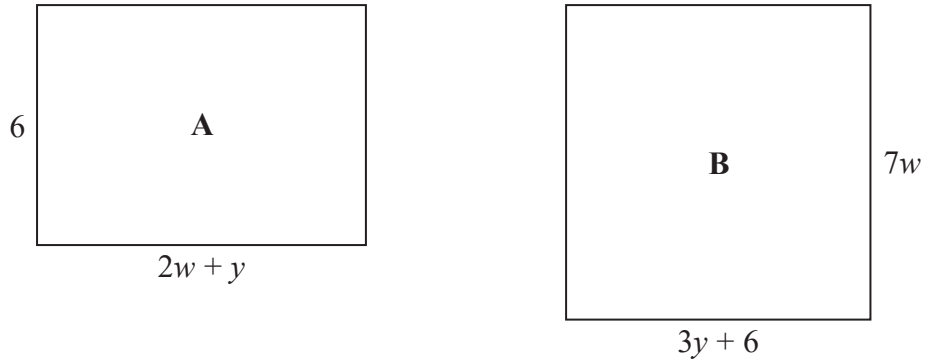
$8x = 54$

$x = \frac{54}{8}$

116 cm²

(Total for Question is 4 marks)

7. The diagram shows two rectangles, **A** and **B**.



All measurements are in centimetres.

The area of rectangle **A** is equal to the area of rectangle **B**.

Find an expression for y in terms of w.

Area of Rectangle A:

$6(2w + y) = 12w + 6y$

Area of Rectangle B:

$7w(3y + 6) = 21wy + 42w$

Make y the subject:

$12w + 6y = 21wy + 42w$ (1)

$6y - 21wy = 42w - 12w$ (1)

$y(6 - 21w) = 30w$ (1)

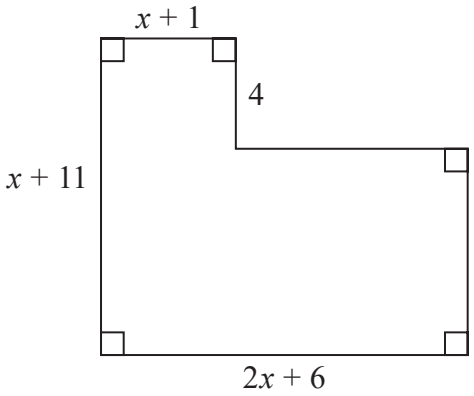
(1)

$\frac{30w}{6 - 21w}$

$\therefore y = \frac{30w}{6 - 21w}$

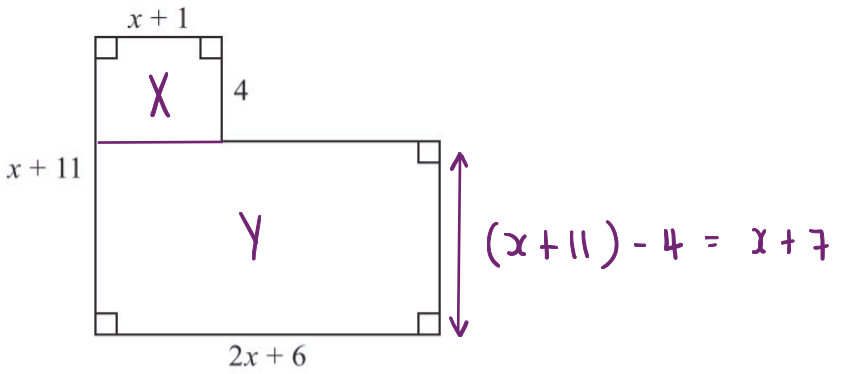
(Total for Question is 4 marks)

8. Here is a shape with all its measurements in centimetres.



The area of the shape is A cm^2

Show that $A = 2x^2 + 24x + 46$



Area of X :

$4(x + 1) = 4x + 4$

Area of Y : (1)

$(2x + 6)(x + 7)$

$= 2x^2 + 14x + 6x + 42$

$= 2x^2 + 20x + 42$

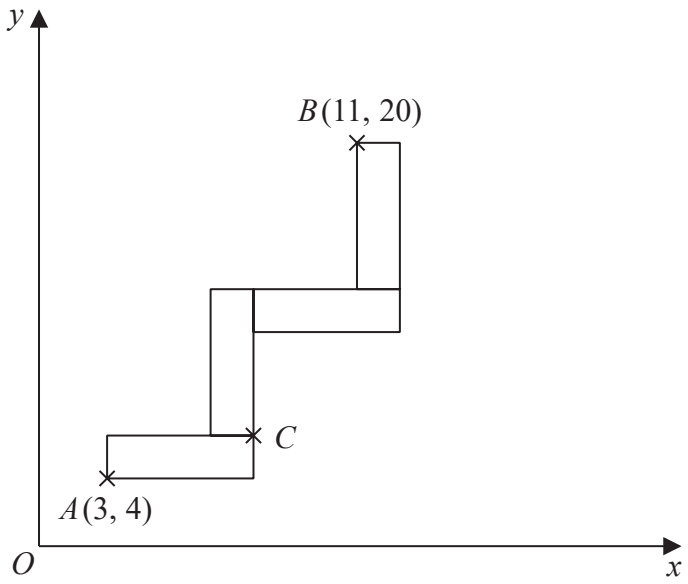
Total area of shape: (1)

$(4x + 4) + (2x^2 + 20x + 42)$

$= 2x^2 + 24x + 46$ (1)

(Total for Question is 3 marks)

9. A pattern is made from four identical rectangles.
The sides of the rectangles are parallel to the axes.



Point A has coordinates (3, 4)
Point B has coordinates (11, 20)
Point C is marked on the diagram.

Work out the coordinates of C.
You must show all your working.

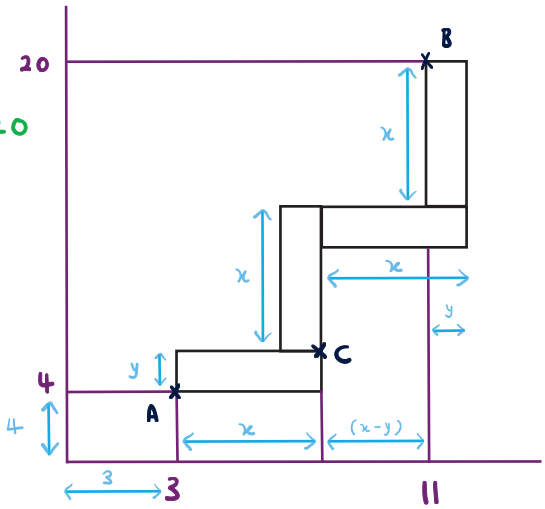
$4 + y + x + x = 20$

$4 + y + 2x = 20$

$2x + y = 16.$

$2x = 16 - y.$

(1)



$3 + x + (x - y) = 11.$

$3 + 2x - y = 11.$

$2x - y = 8.$

(1) $2x = 8 + y.$

$16 - y = 8 + y.$

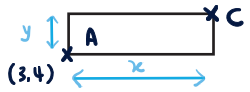
$8 = 2y.$

$y = 4.$ (1)

$2x = 8 + y$

$2x = 8 + 4 = 12.$

$x = 6.$



$C = ((3 + x), (4 + y))$

$= (9, 8)$

(1) (1)

(9 , 8)

(Total for Question is 5 marks)

10. Olivia and Jessica have in total half as many sweets as Fran and Gary have in total.

Fran and Gary share their sweets in the ratio 2 : 3
Olivia and Jessica share their sweets in the ratio 9 : 1

Fran got w sweets.
Gary got x sweets.
Olivia got y sweets.
Jessica got z sweets.

Find, in its simplest form, $w : x : y : z$

Let's say Olivia and Jessica have 50 sweets.
Then Fran and Gary have 100 sweets. (1)

F : G = 2 : 3 → 5 parts for 100 sweets.

∴ 1 part = 20 sweets.

F : G = 2 : 3 = 40 : 60.

O : J = 9 : 1 → 10 parts for 50 sweets. (1)

∴ 1 part = 5 sweets.

O : J = 9 : 1 = 45 : 5.

w : x : y : z
= F : G : O : J (1)
= 40 : 60 : 45 : 5
= 8 : 12 : 9 : 1 (1)

8 : 12 : 9 : 1

(Total for Question is 4 marks)

11. The curve **C** has equation $y = x^2 + 3x - 3$

The line **L** has equation $y - 5x + 4 = 0$

Show, algebraically, that **C** and **L** have exactly one point in common.

If C and L have one point (x, y) in common, they have the same x-value and the same y-value.

© $y = x^2 + 3x - 3$.

Ⓛ $y - 5x + 4 = 0 \therefore y = 5x - 4$.

$\therefore x^2 + 3x - 3 = 5x - 4$. (1)

$x^2 + 3x - 3 = 5x - 4$.

$x^2 - 2x - 3 = -4$.

$x^2 - 2x + 1 = 0$.

$x^2 - 2x + 1 = 0$. (1)

$(x-1)^2 = 0$.

$\therefore x = 1$. (1)

(1) There is only one value of x and so C and L have only one point in common.

(Total for Question is 4 marks)

12. Pat throws a fair coin n times.

Find an expression, in terms of n, for the probability that Pat gets at least 1 head and at least 1 tail.

It is almost certain that Pat will get at least one head and one tail.

The ONLY time this is NOT possible is if there are all heads or all tails.

$P(\text{all heads}) = \left(\frac{1}{2}\right)^n$. $P(\text{all tails}) = \left(\frac{1}{2}\right)^n$

$P(\text{all heads OR all tails}) = \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n$ (1)

$P(\text{at least one head and one tail})$

$= 1 - (\text{all heads or all tails})$

$= 1 - \left(\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n\right)$

$= 1 - \left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n$ (1)

(Total for Question is 2 marks)